

Local-Duality QCD Sum Rules for Pion Elastic and $(\pi^0, \eta, \eta') \rightarrow \gamma\gamma^*$ Transition Form Factors Revisited

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The local-duality formulation of QCD sum rules allows for the prediction of hadronic form factors without knowledge of the subtle details of their structure. With the aid of this formalism, we take a fresh look at the behaviours of the charged-pion elastic form factor and of the form factors entering in the transitions of the ground-state neutral unflavoured pseudoscalar mesons π^0, η, η' to one real and one virtual photon within a broad range of momentum transfers Q^2 . The uncertainties induced by the approximations inherent to this local-duality approach are estimated by studying, in parallel to QCD, quantum-mechanical potential models, where the exact form factors, obtained by solving the Schrödinger equation, may be compared with the corresponding local-duality sum-rule results. For $Q^2 \geq 5-6 \text{ GeV}^2$, we judge the predictions of the simplest local-duality model to be reliable and expect their accuracy to improve very fast with increasing Q^2 . The large- Q^2 prediction for the pion elastic form factor should be approached already at moderate momentum transfer $Q^2 \approx 4-8 \text{ GeV}^2$; large deviations from its local-duality behaviour for $Q^2 = 20-50 \text{ GeV}^2$, predicted by some hadron-structure models, seem rather unlikely. The $(\eta, \eta') \rightarrow \gamma\gamma^*$ form factors deduced from the simplest local-duality approach exhibit excellent agreement with experiment. In startling contrast, BABAR measurements of the $\pi^0 \rightarrow \gamma\gamma^*$ form factor imply local-duality violations which even rise with Q^2 .

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1. Introduction: Motivation and Incentive for Reconsidering a Long-Standing Issue

QCD sum rules aim to predict the characteristic features of ground-state hadrons (their masses, decay constants, form factors, etc.) from the underlying quantum field theory of strong interactions, quantum chromodynamics (QCD), by evaluating matrix elements of suitably chosen operators both on the level of hadrons and on the level of the QCD degrees of freedom quarks and gluons. Wilson's *operator product expansion* allows for the conversion of these *nonlocal* operators into series of local operators. By this process the QCD-level matrix elements receive both perturbative contributions as well as non-perturbative contributions involving universal quantities called vacuum condensates. In order to *suppress* the contributions of hadronic excitations and continuum and to *remove* subtraction terms, *Borel transformations* to new variables, dubbed as the Borel mass parameters, are performed. Representing the perturbative contributions to our QCD-level matrix elements in form of dispersion integrals over corresponding spectral densities allows us to bypass our ignorance about higher states by invoking the concept of *quark-hadron duality*: beyond some *effective thresholds* the perturbative QCD contributions and the expressions of hadron excitations and continuum are assumed to cancel. The outcome of these steps are sum rules relating QCD parameters to observable hadron properties. In the limit of *infinitely large* Borel mass parameters, all non-perturbative QCD contributions vanish and we are left with what is known as local-duality (LD) form of QCD sum rules, rendering possible to derive features of ground-state hadrons from perturbative QCD and our effective-threshold ideas.

Recently, we applied the LD sum-rule formalism to reanalyze both the elastic form factor of the pion [1] and the form factor that describes the transition $P \rightarrow \gamma\gamma^*$ of some light neutral pseudoscalar meson $P = \pi^0, \eta, \eta'$ to a real photon γ and a virtual photon γ^* [2]. One particularly attractive feature of the LD sum-rule approach is the possibility to extract predictions for hadron form factors without knowledge of all subtle details of the structure of the hadronic bound states and to consider different hadrons on an equal footing. Here, we take a retrospective look from bird's eye view at our findings: After recalling, for the example of the pion, the rather well-known basic features of the LD sum-rule approach to pseudoscalar-meson form factors, in order to get an idea (or even rough estimate) of the accuracy to be expected for real-life mesons described by QCD sum rules we make a brief and in the meanwhile well-established sidestep to their quantum-mechanical analogues as a means to examine the uncertainties induced by modeling the impact of higher hadronic states in a rather naïve fashion. Then, equipped with sufficient confidence in the reliability of the adopted LD approximation for the effective thresholds, we discuss, in turn, the π elastic and $(\pi^0, \eta, \eta') \rightarrow \gamma\gamma^*$ transition form factors.

2. Dispersive Three-Point QCD Sum Rules in the Limit of Local Duality [3]

The basic objects exploited here for the investigation of the behaviour of form factors $F(Q^2)$ as functions of the involved momentum transfers squared, $Q^2 = -q^2 \geq 0$, are *three-point functions*, the vacuum correlator of one vector and two axialvector currents, with *double spectral density* Δ_{pert} , for the elastic form factor $F_\pi(Q^2)$ and the vacuum correlator of one axialvector and two vector currents, with *single spectral density* σ_{pert} , for the transition form factor $F_{\pi\gamma}(Q^2)$, satisfying the LD sum rules

$$F_\pi(Q^2) = \frac{1}{f_\pi^2} \int_0^{s_{\text{eff}}(Q^2)} ds_1 \int_0^{s_{\text{eff}}(Q^2)} ds_2 \Delta_{\text{pert}}(s_1, s_2, Q^2), \quad F_{\pi\gamma}(Q^2) = \frac{1}{f_\pi} \int_0^{\bar{s}_{\text{eff}}(Q^2)} ds \sigma_{\text{pert}}(s, Q^2). \quad (2.1)$$

Here, f_π is the charged-pion decay constant: $f_\pi = 130$ MeV. Now all details of the non-perturbative dynamics are encoded in the effective thresholds $s_{\text{eff}}(Q^2)$ and $\bar{s}_{\text{eff}}(Q^2)$ that enter as upper endpoints.

We take the liberty of introducing the notion of an *equivalent effective threshold*, defined by the requirement that the use of this quantity as effective threshold in the appropriate dispersive sum rule — such as the LD representatives of Eq. (2.1) — reproduces for the form factor under consideration either given experimental data or a particular theoretical prediction exactly. With such powerful tool at our disposal, we are able to quantify our observations and make our conclusions much more clear.

Within perturbation theory, the spectral densities $\Delta_{\text{pert}}(s_1, s_2, Q^2)$ and $\sigma_{\text{pert}}(s, Q^2)$ are derived as series expansions in powers of the strong coupling α_s by evaluating the relevant Feynman diagrams:

$$\begin{aligned}\Delta_{\text{pert}}(s_1, s_2, Q^2) &= \Delta_{\text{pert}}^{(0)}(s_1, s_2, Q^2) + \alpha_s(Q^2) \Delta_{\text{pert}}^{(1)}(s_1, s_2, Q^2) + O(\alpha_s^2), \\ \sigma_{\text{pert}}(s, Q^2) &= \sigma_{\text{pert}}^{(0)}(s, Q^2) + \alpha_s(Q^2) \sigma_{\text{pert}}^{(1)}(s, Q^2) + O(\alpha_s^2).\end{aligned}\quad (2.2)$$

As far as their aspects relevant for our present purposes are concerned, the theoretical status of these spectral densities may be summarized as follows. In the double spectral density $\Delta_{\text{pert}}(s_1, s_2, Q^2)$, for fixed $s_{1,2}$ and large momentum transfers Q^2 , the one-loop contribution $\Delta_{\text{pert}}^{(0)}(s_1, s_2, Q^2)$ vanishes like $\Delta_{\text{pert}}^{(0)}(s_1, s_2, Q^2) \propto 1/Q^4$ and the two-loop contribution $\Delta_{\text{pert}}^{(1)}(s_1, s_2, Q^2)$ approaches the behaviour [4]

$$\Delta_{\text{pert}}^{(1)}(s_1, s_2, Q^2) \xrightarrow{Q^2 \rightarrow \infty} \frac{1}{2\pi^3 Q^2};$$

in other words, in the limit $Q^2 \rightarrow \infty$ the lowest-order term decays faster than the next-to-lowest term. In the single spectral density $\sigma_{\text{pert}}(s, Q^2)$, the two-loop correction $\sigma_{\text{pert}}^{(1)}(s, Q^2)$ has been proven [5] to vanish identically: $\sigma_{\text{pert}}^{(1)}(s, Q^2) \equiv 0$. Higher-order radiative corrections have not yet been calculated.

With the required spectral densities available at least up to some order of perturbation theory, as soon as the dependencies of the effective thresholds $s_{\text{eff}}(Q^2)$ and $\bar{s}_{\text{eff}}(Q^2)$ on the momentum transfer Q^2 have been found, the form factors of interest can be easily extracted from the LD sum rules (2.1). Factorization theorems for hard form factors [6], allowing for separation of the dynamics into short- and long-distance contributions, establish the asymptotic behaviour of the form factors for large Q^2 :

$$Q^2 F_\pi(Q^2) \xrightarrow{Q^2 \rightarrow \infty} 8\pi \alpha_s(Q^2) f_\pi^2, \quad Q^2 F_{\pi\gamma}(Q^2) \xrightarrow{Q^2 \rightarrow \infty} \sqrt{2} f_\pi.$$

The sum rules (2.1) with the spectral functions (2.2) reproduce, at $O(\alpha_s^2)$ accuracy, this behaviour if

$$\lim_{Q^2 \rightarrow \infty} s_{\text{eff}}(Q^2) = \lim_{Q^2 \rightarrow \infty} \bar{s}_{\text{eff}}(Q^2) = 4\pi^2 f_\pi^2 \approx 0.671 \text{ GeV}^2 \quad (2.3)$$

holds. The remaining task is to determine the behaviour of the effective thresholds at finite values of Q^2 . Unfortunately, as analyzed in detail in Refs. [7], the formulation of a reliable criterion for fixing a threshold poses a somewhat delicate problem as, for finite Q^2 , the effective thresholds $s_{\text{eff}}(Q^2)$ and $\bar{s}_{\text{eff}}(Q^2)$ cannot be assumed to be equal to their asymptotes (2.3); rather, they will depend on Q^2 and, generally, differ from each other [8]. A very simple idea is to assume that the use of their asymptotic values provides a meaningful approximation also at moderate but not too small momentum transfer: $s_{\text{eff}}(Q^2) = \bar{s}_{\text{eff}}(Q^2) = 4\pi^2 f_\pi^2$. This choice defines a straightforward albeit rather naïve LD model [3]. It goes without saying that such crude approximations to the effective thresholds may be well suited to reproduce the overall trend but can hardly account for any subtle detail of confinement dynamics.

3. Exact and Local-Duality Form Factors in Quantum-Mechanical Potential Models

The (quantum-field-theoretic) LD sum-rule approach to bound-state form factors may be easily carried over to quantum mechanics. Within the latter framework, the features of any bound state can be obtained with, in principle, arbitrarily high precision from the related solution of the Schrödinger equation for the Hamiltonian governing the dynamics of the system under consideration. Therefore, quantum-mechanical potential models constitute an ideal test ground for estimating the significance of LD models that employ for the effective thresholds entering in the adopted sum rules the constant limits fixed by some asymptotic behaviour at experimentally accessible lower momentum transfers. For this very reason, we examine quantum-mechanical potential models defined by Hamiltonians H which must incorporate, for the study of the elastic form factor, confining *and* Coulomb interactions ($\eta = 1$) but, for the investigation of the transition form factor, merely confining interactions ($\eta = 0$):

$$H = \frac{\mathbf{k}^2}{2m} + V_{\text{conf}}(r) - \eta \frac{\alpha}{r}, \quad V_{\text{conf}}(r) = \sigma_n (mr)^n, \quad r \equiv |\mathbf{x}|, \quad n = 2, 1, 1/2.$$

We ensure a realistic description of mesons by adopting for our numerical analysis parameter values appropriate for hadron physics: $m = 0.175$ GeV for the reduced mass of light constituent quarks and $\alpha = 0.3$ for the coupling strength α of the Coulomb interaction term. For the confining interactions, we consider several power-law potential shapes $V_{\text{conf}}(r)$, adjusting the associated coupling strengths σ_n such that in each case the Schrödinger equation predicts the same value $\psi(0) = 0.078$ GeV^{3/2} for the ground-state wave function ψ at the origin: $\sigma_2 = 0.71$ GeV, $\sigma_1 = 0.96$ GeV and $\sigma_{1/2} = 1.4$ GeV. Then, the size of the lowest-lying bound state is about 1 fm and thus of typical hadronic dimensions.

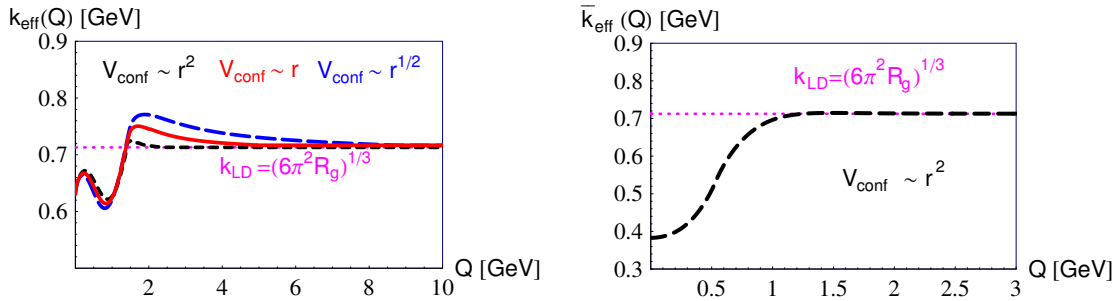


Figure 1: Exact quantum-mechanical effective thresholds for elastic (left) and transition (right) form factors.

With the numerically exact solution of the Schrödinger equation at hand, we are in a position to confront the form factors arising thereof with corresponding predictions of the quantum-mechanical counterparts of the LD QCD sum rules (2.1), which involve effective thresholds $k_{\text{eff}}(Q)$ and $\bar{k}_{\text{eff}}(Q)$, respectively. As in the QCD case, the asymptotic behaviour of the elastic and transition form factors in the limit of infinitely large momentum transfer Q may be derived from factorization theorems [6]. In terms of the ground-state decay constant $R_g \equiv |\psi(0)|^2$, this asymptotic behaviour is guaranteed if the effective thresholds fulfill $k_{\text{eff}}(Q \rightarrow \infty) = \bar{k}_{\text{eff}}(Q \rightarrow \infty) = (6\pi^2 R_g)^{1/3}$. Figure 1 shows that the LD model $k_{\text{eff}}(Q) = \bar{k}_{\text{eff}}(Q) = (6\pi^2 R_g)^{1/3}$ approximates independently of the confining potential in use the exact effective thresholds yielding the true form factors with improving accuracy, starting for the elastic form factor at $Q^2 \approx 5\text{--}8$ GeV² and for the transition form factor at some even lower Q^2 value.

4. The Pion Elastic Form Factor [1]

The pion belongs, beyond doubt, to the best-studied mesons. Nevertheless, the one or the other of its most important properties still cannot seriously be claimed to be sufficiently well understood.¹ Figure 2 displays a snapshot of the present status of the pion's electromagnetic or elastic form factor $F_\pi(Q^2)$ from both the experimental [9] and the theoretical [1, 10] points of view. Obviously, there is ample room for controversy, but no consensus on $F_\pi(Q^2)$ for momentum transfers $Q^2 \approx 5\text{--}50 \text{ GeV}^2$.

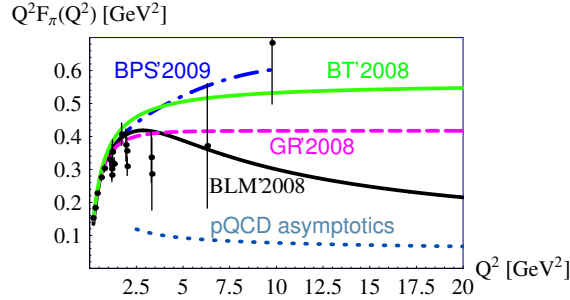


Figure 2: Pion elastic form factor $F_\pi(Q^2)$: experimental data [9] and some recent theoretical findings [1, 10].

In order to cast some light onto these disquieting puzzles, Figure 3 depicts our translation of the findings summarized in Fig. 2 to equivalent effective thresholds $s_{\text{eff}}(Q^2)$ calculated back from either experimental data or theoretical predictions for $F_\pi(Q^2)$: the exact effective threshold extracted from the data is compatible with the assumption that the LD limit is approached at rather low Q^2 whereas, contrary to quantum physics, theory seems not to care about local duality, at least for $Q^2 \leq 20 \text{ GeV}^2$.

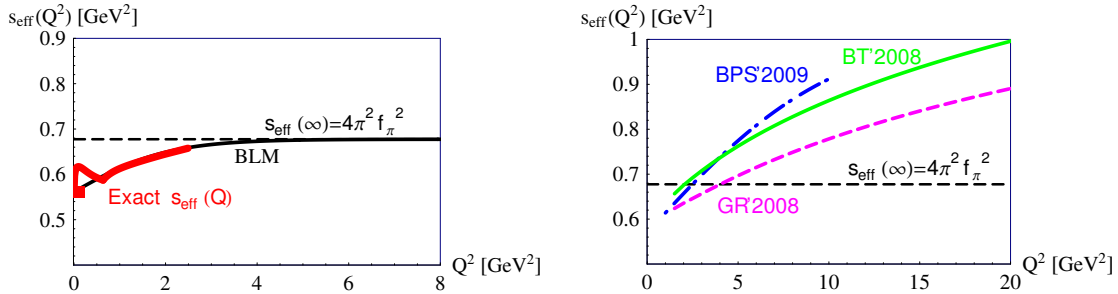


Figure 3: Parametrization of the effective threshold $s_{\text{eff}}(Q^2)$ by an improved LD model [1] (labelled BLM) vs. exact behaviour (red) of the equivalent effective threshold extracted from experimental data [9] (left), and equivalent effective thresholds corresponding to the theoretical results for F_π [1, 10] depicted in Fig. 2 (right).

Rather precise measurements may be expected from JLab after the 12 GeV upgrade of CEBAF.

¹Of course, whenever some problem in the treatment of any of the ground-state pseudoscalar mesons is encountered, as a kind of automatic reflex-like response one may be tempted to blame within QCD the pseudo-Goldstone-boson nature of the particle for preventing us from acquiring a satisfactory understanding. Nevertheless, all comprehensive approaches should be expected to be able to deal with this sort of “inconvenience” and to ultimately incorporate such crucial features.

5. The $(\pi^0, \eta, \eta') \rightarrow \gamma\gamma^*$ Transition Form Factors [2]

In order to consolidate our concerns and to substantiate our confusions, we discuss the η and η' transitions $(\eta, \eta') \rightarrow \gamma\gamma^*$ before turning to the controversial issue of the pion's transition $\pi^0 \rightarrow \gamma\gamma^*$.

5.1 Form Factors for the Transitions $(\eta, \eta') \rightarrow \gamma\gamma^*$

The two isoscalar mesons η and η' , having the same J^{PC} quantum numbers, are mixtures of *all* light quarks. In the flavour basis, the mixing of the non-strange and strange contributions is given by

$$|\eta\rangle = \left| \frac{\bar{u}u + \bar{d}d}{\sqrt{2}} \right\rangle \cos \phi - |\bar{s}s\rangle \sin \phi, \quad |\eta'\rangle = \left| \frac{\bar{u}u + \bar{d}d}{\sqrt{2}} \right\rangle \sin \phi + |\bar{s}s\rangle \cos \phi,$$

with mixing angle $\phi \approx 39.3^\circ$; see, *e.g.*, Refs. [11, 12]. The form factors reflect this flavour structure:

$$F_{\eta\gamma}(Q^2) = \frac{5F_{n\gamma}(Q^2)}{3\sqrt{2}} \cos \phi - \frac{F_{s\gamma}(Q^2)}{3} \sin \phi, \quad F_{\eta'\gamma}(Q^2) = \frac{5F_{n\gamma}(Q^2)}{3\sqrt{2}} \sin \phi + \frac{F_{s\gamma}(Q^2)}{3} \cos \phi.$$

Here, the non-strange and $(\bar{s}s)$ components $F_{n\gamma}(Q^2)$ and $F_{s\gamma}(Q^2)$ of the LD form factors are given by

$$F_{n\gamma}(Q^2) = \frac{1}{f_n} \int_0^{\bar{s}_{\text{eff}}^{(n)}(Q^2)} ds \sigma_{\text{pert}}^{(n)}(s, Q^2), \quad F_{s\gamma}(Q^2) = \frac{1}{f_s} \int_{4m_s^2}^{\bar{s}_{\text{eff}}^{(s)}(Q^2)} ds \sigma_{\text{pert}}^{(s)}(s, Q^2),$$

where $\sigma_{\text{pert}}^{(n)}$ and $\sigma_{\text{pert}}^{(s)}$ label the single spectral density σ_{pert} of Eq. (2.1) with the corresponding quark, $n = u, d$ or s , propagating in the loop; each component utilizes an effective threshold of its own [12]:

$$\bar{s}_{\text{eff}}^{(n)} = 4\pi^2 f_n^2, \quad f_n \approx 1.07 f_\pi, \quad \bar{s}_{\text{eff}}^{(s)} = 4\pi^2 f_s^2, \quad f_s \approx 1.36 f_\pi.$$

In our numerical calculations, we adopt $m_u = m_d = 0$ and $m_s = 100$ MeV for the light-quark masses.

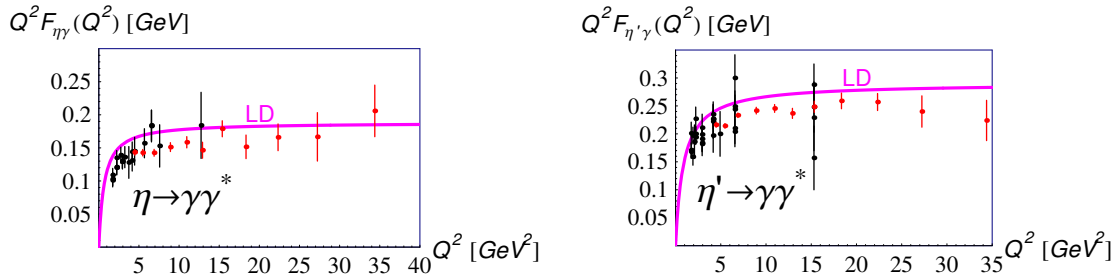


Figure 4: Transition form factors $F_{(\eta, \eta')\gamma}(Q^2)$: for η and η' the LD model fits the experimental data [13, 14].

According to all our experience gained by in-depth investigations of the LD sum-rule approach within quantum mechanics, this straightforward but admittedly not too sophisticated LD framework may not perform really well for low momentum transfers Q^2 , where, as a brief look at Fig. 3 reveals, the exact effective threshold is below the constant LD effective threshold inferred from the large- Q^2 form-factor behaviour. However, for larger momentum transfer the simple quantum-mechanical LD model entails accurate predictions for form factors. Figure 4 shows that, for both η and η' transition form factors, we find the anticipated overall agreement between the LD predictions and experiment.

5.2 Form Factor for the Transition $\pi^0 \rightarrow \gamma\gamma^*$

In view of the undeniable successes of the LD model in the case of the π elastic form factor and of the η and η' transition form factors, its failure in the case of the π^0 transition form factor $F_{\pi\gamma}(Q^2)$ is all the more surprising. Figure 5 displays how markedly the LD prediction for $F_{\pi\gamma}(Q^2)$ misses the BABAR data [15]. This becomes even more manifest by the linear rise with Q^2 of the corresponding equivalent effective threshold $\bar{s}_{\text{eff}}(Q^2)$, which, at least in the region up to $Q^2 \approx 40 \text{ GeV}^2$, exhibits no tendency of approaching its LD limit (2.3). This intriguing puzzle still awaits a compelling solution.

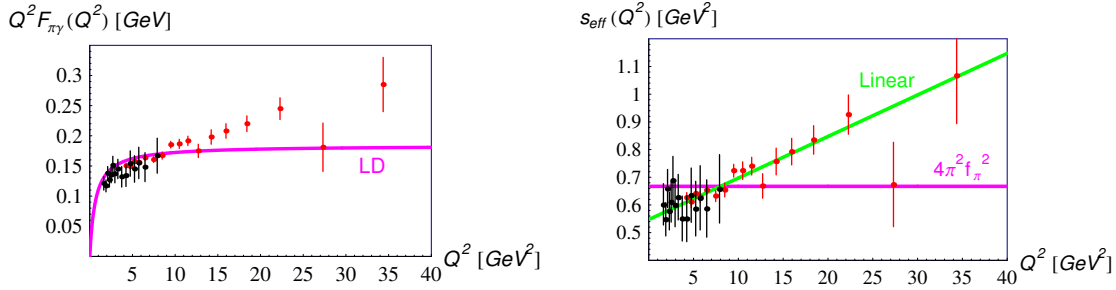


Figure 5: Form factor $F_{\pi\gamma}(Q^2)$ for the pion transition $\pi^0 \rightarrow \gamma\gamma^*$: some experimental data [13, 15], at least the BABAR data (red dots), apparently diverge from our LD prediction; this unexpected behaviour is reflected by the equivalent effective threshold $\bar{s}_{\text{eff}}(Q^2)$ exhibiting a linear rise with Q^2 instead of approaching its LD limit.

6. Summary: Findings and Conclusions

By reconsidering the dependence of the pion elastic [1] and π^0, η, η' transition [2] form factors on the momentum transfer Q^2 using QCD sum rules in LD limit, we gain highly interesting insights:

Pion elastic form factor: Transferring the outcomes of our quantum-mechanical analysis to QCD, we expect the simple LD model to be applicable with increasing accuracy for $Q^2 \geq 4\text{--}8 \text{ GeV}^2$ irrespective of the adopted confining interactions. For realistic confining interactions, this LD model reproduces the elastic form factor for $Q^2 \geq 20\text{--}30 \text{ GeV}^2$ with high precision. Accurate measurements of this form factor at small Q^2 suggest that assuming for the effective threshold its LD limit already at rather low $Q^2 = 5\text{--}6 \text{ GeV}^2$ may constitute a reasonable approximation. Hence, large deviations from this LD limit at $Q^2 = 20\text{--}50 \text{ GeV}^2$ must be regarded as unlikely.

Transition form factors for π^0, η, η' : Our observations in quantum mechanics can be understood as hints that, for bound states of typical hadron extensions, the LD approach should work well for Q^2 larger than a few GeV^2 , and it indeed does for the $\eta \rightarrow \gamma\gamma^*$ and $\eta' \rightarrow \gamma\gamma^*$ form factors. However, a recent measurement of the form factor for the neutral-pion transition $\pi^0 \rightarrow \gamma\gamma^*$ by the BABAR experiment [15] implies a violation of local duality which even grows with Q^2 , at least up to Q^2 as high as 40 GeV^2 . Within the LD sum-rule formalism (2.1), such behaviour of a transition form factor cannot be accommodated by a *constant* equivalent effective threshold but must be described by a linear Q^2 -dependence of $\bar{s}_{\text{eff}}(Q^2)$; a convincing explanation of this has yet to be found. This conclusion enjoys full agreement with the findings of Refs. [16, 17].

Acknowledgments

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